Model theory, list 8.

$\delta = \delta(x, y), \varphi, \psi$ always denote formulas of L.

1. Assume that $\varphi(x, y)$ i $\psi(x, y)$ are stable. Prove that $\neg\varphi(x, y)$, $\varphi \wedge \psi(x, y)$ and $\chi(y, x) = \varphi(x, y)$ are also stable.

2. For types p, q prove that:
   (a) $(p \vee q)(\mathcal{M}) = p(\mathcal{M}) \cup q(\mathcal{M})$,
   (b)$\exists x p(x, y)(\mathcal{M}) = \{b \in \mathcal{M} : \exists a \in \mathcal{M} \models p(a, b)\}$.
   (c) $\forall x p(x, y)(\mathcal{M}) = \{b \in \mathcal{M} : \forall a \in \mathcal{M} \models p(a, b)\}$.

3. Let $p \in S_\delta(M)$. Prove that $R_\delta(p) = CB_\delta(p)$.

4. Assume that $\chi(y)$ is a δ-definition of type $p \in S_\delta(M)$ and $M \prec N$. Prove that there is a unique type $q \in S_\delta(N)$ such that χ is a δ-definition of q.

5. Assume that T is stable, $p \in S(M)$.
   (a) Prove that there is a unique type $q \in S(\mathcal{M})$ such that for every $\delta(x, y)$, the δ-definition of type p is the δ-definition of type q.
   For $A \supseteq M$ we define $p_A = q|A$.
   (b) Assume U is an ultrafilter on M (that is, in the algebra of all subsets of M) such that $\{\varphi(M) : \varphi \in p\} \subseteq U$. Prove that for every $\psi(y) \in q$, $\psi(\mathcal{M}) \cap M \in U$. (That means that for stable T, the type p_A corresponds to the type from the problem from list 5 and does not depend on the choice of U).
   (c) Prove that $R_\delta(p) = R_\delta(q)$ oraz $R_{\delta,2}(p) = R_{\delta,2}(q)$, for every $\delta(x, y)$.