Model Theory, list 7.

1. Prove that if $a \in acl(Ab)$, then $RM(a/A) \leq RM(b/A)$.

2. Prove that $tp(ab/A)$ is s-isolated $\iff$ $tp(a/A)$ is s-isolated and $tp(b/Aa)$ is s-isolated.

3. Assume that $p \in S(A)$, $RM(p) < \infty$ i $A \subseteq B$. Prove that

$$Mlt(p) = \sum \{Mlt(q) : p \subseteq q \in S(B) \text{ i } RM(q) = RM(p)\}$$

(in particular over $B$ there are $\leq Mlt(p)$ complete extensions of type $p$ of the same Morley rank).

4. Assume that $T$ has no Vaughtian pair. Prove that for every $\varphi(x, \overline{y}) \in L$ there is $n < \omega$ such that for every $\overline{a} \subset \mathcal{M}$, $|\varphi(\mathcal{M}, \overline{a})| < \aleph_0 \Rightarrow |\varphi(\mathcal{M}, \overline{a})| < n$. (hint: if there is no such $n$, consider a theory describing a Vaughtoan trople $(M, N, \varphi(x, \overline{c})$, usung a new predicate symbol and new constant symbol).

5. Assume that $T$ is $\aleph_0$-stable. Let $M$ be a prime model of $T$.
(a) Prove that there is a non-algebraic formula $\varphi(x) \in L(M)$ such that for every formula $\psi(x) \in L(M)$, one of the formulas $\varphi \wedge \psi$, $varphi \wedge \neg\psi$ is algebraic. (hint: consider a type $p \in S(M)$ of Cantor-Bendixson rank 1.)
(b) Assuming additionally that $T$ has no Vaughtian pair prove that the formula $\varphi$ from point (a) is strongly minimal. (hint: use the previous problem).

6. Prove that if $M$ is $\aleph_0$-saturated, $p \in S(M)$ and $RM(p) < \infty$, then $Mlt(p) = 1$. (hint: the assumption of $\aleph_0$-saturation may be omitted, but the proof is rather hard, later).

7. Assume that $A, B \subset \mathcal{M}$ and $f : A \to B$ is an elementary surjection. Prove that $f$ extends to an elementary function $f' : acl(A) \to acl(B)$. Prove that $f'$ is onto. Is $f'$ unique?