Model theory, list 5.

1. Assume that $A \subseteq B$.
(a) Prove that if $p \in S(A)$ is non-algebraic, then p extends to a non-algebraic type in $S(B)$.
(b)* Assume that $a_1, \ldots, a_n \notin acl(A)$. Prove that here exist $b_1, \ldots, b_n \notin acl(B)$ such that $tp(a_1 \ldots a_n/A) = tp(b_1 \ldots b_n/A)$.

2. (a) Prove that every algebraic type $p \in S(A)$ is isolated.
(b) Give an example of a theory T and an isolated complete type that is not algebraic.
(c) Prove that a comlete type over a model M is isolated $\iff$ it is algebraic.
(d) Assume that $p \in S(\emptyset)$ and $p(M)$ is non-empty and finite. Does it follow that p is algebraic?

3. Prove that in a compact 0-dimensional space X (that is, a compact space where clopen sets form a basis of topology) for $p \in X$, $CB(p) = min\{CB(U) : p \in U \in Clopen(X)\}$.

4. Assume that $a \in dcl(A \cup \{b\})$. Prove (by induction, of course) that $CB(a/A) \leq CB(b/A)$. (hint: Prove that there are A-definable sets U and V with $b \in U$ and $a \in V$, and an A-definable function $f : U \to V$ with $f(b) = a$.) Comment: the conclusion holds also when $a \in acl(A \cup \{b\})$, but it is harder.

5. Prove that if $tp(a) = tp(b)$, then $RM(\varphi(x, a)) = RM(\varphi(x, b))$.

6. Prove that the set of values if Morley rank (in a fixed theory T) is an initial segment of Ord, possibly augmented by ∞.

7. Prove that in an ω-stable theory, $RM(x = x)$ is countable (recall that we assume that the language is countable) .

8. * Prove that if $RM(x = x)$ is finite, then $RM(x_0 = x_0 \wedge x_1 = x_1)$ is also finite (the point is that the set of realizations of the first formula is the whole model M, while of the second one the set $M \times M$).

9. * Niech $T = Th(2^\omega, +, 0, P_n)_{n<\omega}$, where $(f + g)(n) = f(n) + g(n) (\mod 2)$ and $P_n(f)$ holds iff $f(n) = 0$. Prove that T has no prime model, and it has $2^{\aleph_0}$ minimal models. (hint: prove that T is q.e., write down the axioms and describe the models of T)