Model Theory, List 1.

1. Assume that $A \subseteq B \subseteq M$.
(a) Prove that the restriction function $r : S_n(B) \to S_n(A)$ is continuous.
(b) Prove that the restriction function $r' : S_{n+1}(A) \to S_n(A)$ induced by restriction to formulas in free variables $x_0, \dots, x_{n-1}$, is continuous.

2. Prove that if M is κ-saturated, $p \in S_n(A)$, $A \subseteq M$, $|A| < \kappa$, then p is realized in M.

3. Prove that if $M \subseteq N$ (substructure), then for every term $\tau(\overline{x})$ of language L and every $\overline{a} \subseteq M$ we have that $\tau^M(\overline{a}) = \tau^N(\overline{a})$.

4. (a) Verify correctness of the construction of direct limit $(M_\infty, f_{i\infty})_{i\in I}$ of an elementary directed system of strictures.
Prove that (b) this structure satisfies the definition of the direct limit and (c) the functions $f_{i\infty}$ are elementary.

5. We say that two directed systems $(M_i, f_{ij})_{i\leqslant j\in I}$, $(N_i, g_{ij})_{i\leqslant j\in I}$ are isomorphic if there is a family of isomorphisms $h_i : M_i \to N_i, i \in I$, commuting with the connecting functions f_{ij}, g_{ij}. Prove that every directed system of structures is isomorphic to a system, where all connecting functions are inclusions. Direct limit of such a system is just the usual union.

6. Present arbitrary structure (for a countable language) as the direct limit of an elementary directed system of countable structures.

7. Prove the lemma on extending elementary mappings (from Lecture 2).