Logic R, list 1.

1. Which sentences (formulas) are true in the empty structure?

2. Prove that if a formula φ is an instance of a propositional formula α, then $\models \alpha$ implies $\models \varphi$.

3. Prove that if $\models \alpha \to \beta$ and $p_1, \ldots, p_k$ are all common propositional variables of the propositional formulas α i β, then there is a propositional formula $\gamma(p_1, \ldots, p_k)$ such that $\models \alpha \to \gamma$ and $\models \gamma \to \beta$.

4. Prove that if $v, v' : S \to \{0,1\}$ are logical valuations and $v|_Z = v'|_Z$, then $v = v'$ (Z is the set of propositional variables in S).

5. We define logical connectives $\perp$ and $|$ by $\alpha \perp \beta = \neg(\alpha \wedge \beta),\ \alpha|\beta = \neg(\alpha \vee \beta)$. Prove that using each of these connectives we can define all the other logical connectives; next show that $\perp$ and $|$ are the only connectives with this property (hint: what can the definition of $\neg$ be?).

6–. Fill in randomly the table of logical values of an unknown propositional formula with variables p, q, r. Then write down a formula $\alpha(p, q, r)$ with precisely this table of logical values.

7. Prove that a propositional formula α is equivalent to a formula built from propositional variables and connectives $\wedge, \to$ if and only if for every valuation w, if $w|_Z \equiv 1$, then $w(\alpha) = 1$ (hint: comsider $\neg\alpha$.).

8–. Verify if the cut rule and generalization rule are valid inference rules and if every axiom of KRL is a tautology.

9. Prove the deduction theorem: If φ is a sentence, then $(X \vdash \varphi \to \psi \iff X \cup \{\varphi\} \vdash \psi)$. (hint for $\Leftarrow$: Assume that $\alpha_1, \ldots, \alpha_n = \psi$ is a proof of ψ from $X \cup \{\varphi\}$. Prove by induction that $X \vdash \varphi \to \alpha_i$ for every i). Show that the assumption that φ is a sentence is essential.

10. Prove that the set of formulas X is inconsistent $\iff Cn(X) = Form_L$.

11. Priove the Lindenbaum Theorem: every consistent theory X is contained in some complete consistent theory X' (hint: use Zorna Lemma).

12–. Prove that every formula φ is equivalent to a formula φ' in a normal prenex form, i.e. $\models \varphi \leftrightarrow \varphi'$, where $\varphi' = Q_1 x_{i_1} Q_2 x_{i_2} \ldots Q_k x_{i_k} \psi(x_{i_1}, \ldots, x_{i_k})$ and $Q_i \in \{\forall, \exists\}$, ψ does not contain quantifiers and the scope of each of the quantifiers in φ' is the whole part of φ' to the right of it.

Problems marked with – can not be turned in as homework.