

Ex. 7

Lemat: $PA \vdash S_n + m = n + S_m$

Dowód Ustalamy m . Pokażemy indukcyjnie, że
 $S_n + m = n + S_m$. Gdy $n = 0$:

$PA \vdash S_0 + m$

$$\stackrel{PA5}{=} S(0 + m)$$

$$\stackrel{PA4}{=} S_m$$

$$\stackrel{PA4}{=} 0 + S_m$$

Zet. teraz dla $y < S_n$. Wtedy

$PA \vdash S_n + m$

$$\stackrel{PA5}{=} S(n + m)$$

$$\stackrel{IND}{=} S(m + n)$$

$$\stackrel{PA5}{=} S_m + n$$

$$\stackrel{\text{Lemme}}{=} m + S_n$$

Na mocy P3 mamy $PA \vdash n + m = m + n$.

