

$$\text{Ponad to } 1 - \bar{y} - 2\alpha\bar{x} = 1 - \frac{1-\alpha}{1+\alpha^2} - \frac{2\alpha+2\alpha^2}{1+\alpha^2} = 1 - \frac{1+\alpha+2\alpha^2}{1+\alpha^2}$$

$$= \frac{1+\alpha^2 - 1 - \alpha - 2\alpha^2}{1+\alpha^2} = -\alpha \frac{1+\alpha}{1+\alpha^2} = -\alpha\bar{x}$$

$$-1 + \bar{x} - 2\alpha\bar{y} = -1 + \frac{1+\alpha}{1+\alpha^2} - 2\alpha \frac{1-\alpha}{1+\alpha^2} =$$

$$= -1 + \frac{1+\alpha+2\alpha^2}{1+\alpha^2} = \frac{-d+\alpha^2}{1+\alpha^2} = \alpha \frac{\alpha-1}{1+\alpha^2}$$

$$= -\alpha\bar{y}$$

$$\begin{vmatrix} -\alpha\bar{x} - \lambda & -\bar{x} \\ \bar{y} & -\alpha\bar{y} - \lambda \end{vmatrix} = (\alpha\bar{x} + \lambda)(\alpha\bar{y} + \lambda) + \bar{x}\bar{y} =$$

$$= \alpha^2\bar{x}\bar{y} + \lambda\alpha(\bar{x} + \bar{y}) + \lambda^2 + \bar{x}\bar{y} = 0$$

$$\Delta = \alpha^2(\bar{x} + \bar{y})^2 - 4(\alpha^2 + 1)\bar{x}\bar{y}$$

$$= \alpha^2(\bar{x} - \bar{y})^2 - 4\bar{x}\bar{y} =$$

$$\text{Pytanie: } = \alpha^2 \left(\frac{1+\alpha - 1+\alpha}{1+\alpha^2} \right)^2 - 4 \frac{1-\alpha^2}{(1+\alpha^2)^2}$$

$$= \alpha^2 \frac{(2\alpha)^2}{(1+\alpha^2)^2} - 4 \frac{1-\alpha^2}{(1+\alpha^2)^2}$$

$$= \frac{4\alpha^4 + 4\alpha^2 - 4}{(1+\alpha^2)^2} = \frac{4(\alpha^4 + \alpha^2 - 1)}{(1+\alpha^2)^2}$$

$$\text{Pytanie: Czy } \operatorname{Re} \left(\frac{-\alpha(\bar{x} + \bar{y}) \pm \frac{2}{1+\alpha^2} \sqrt{\alpha^4 + \alpha^2 - 1}}{2} \right) < 0 \quad ?$$

Wolfram tw. że to prawda dla $\alpha \in (0, 1)$.

Dla $\alpha = 0$ podstawowa wersja lotki-Volterra.