$$\begin{cases} x' = f(x,y) = x(1 - y - \alpha x) \\ y' = g(x,y) = -y(1 - x + \alpha y) \end{cases}, \qquad \begin{matrix} x \geq 0 \\ y \geq 0 \end{matrix}$$

Punkty stacjonarne: $x \equiv c_1$, $y \equiv c_2$. Zał. $c_1 \neq 0$, $c_2 \neq 0$

$$\begin{cases} 1 - y - \alpha x = 0 \to y = 1 - \alpha x \\ 1 - x + \alpha y = 0 \to 1 - x + \alpha - \alpha^2 x = 0 \to \frac{1+\alpha}{1+\alpha^2} = x \end{cases}$$

$$y = 1 - \frac{\alpha(1+\alpha)}{1+\alpha^2} = \frac{1-\alpha}{1+\alpha^2}$$

Dla $c_1 = 0 \to 1 + \alpha y = 0 \to y = -\frac{1}{\alpha}$

Dla $c_2 = 0 \to 1 - \alpha x = 0 \to x = \frac{1}{\alpha}$

Oczywiście $c_1 = c_2 = 0$. Zatem pkt. stacjonarne to

$$\underbrace{\left(\frac{1+\alpha}{1+\alpha^2}, \frac{1-\alpha}{1+\alpha^2}\right)}_{(\bar{x},\, \bar{y})}, \quad (0,0), \quad \left(\frac{1}{\alpha}, 0\right), \quad \left(0, -\frac{1}{\alpha}\right)$$

Nie interesuje nas

Niech $\varphi(t) = x(t) - \bar{x}$, $\psi(t) = y(t) - \bar{y}$. Linearyzujemy.

$$\varphi' = f(\varphi + \bar{x}, \psi + \bar{y}) \cancel{- f(\bar{x}, \bar{y})}$$

$$\psi' = g(\varphi + \bar{x}, \psi + \bar{y})$$ Ze wzoru Taylora

$$\begin{pmatrix} \varphi' \\ \psi' \end{pmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x}(\bar{x},\bar{y}) & \frac{\partial f}{\partial y}(\bar{x},\bar{y}) \\ \frac{\partial g}{\partial x}(\bar{x},\bar{y}) & \frac{\partial g}{\partial y}(\bar{x},\bar{y}) \end{bmatrix} + R(\varphi,\psi) = \begin{bmatrix} 1 - \bar{y} - 2\alpha\bar{x} & -\bar{x} \\ \bar{y} & -1 + \bar{x} - 2\alpha\bar{y} \end{bmatrix} + R(\varphi,\psi)$$