$$\begin{cases} x' = f(x,y) = x(1-y-\alpha x) \\ y' = g(x,y) = -y(1-x+\alpha y) \end{cases}, \qquad \begin{matrix} x \geq 0 \\ y > 0 \end{matrix}$$

Punkty stacjonarne: $x \equiv c_1$, $y \equiv c_2$. Zał. $c_1 \neq 0$, $c_2 \neq 0$

$$\begin{cases} 1 - y - \alpha x = 0 \rightarrow y = 1 - \alpha x \\ 1 - x + \alpha y = 0 \rightarrow 1 - x + \alpha - \alpha^2 x = 0 \rightarrow \frac{1+\alpha}{1+\alpha^2} = x \end{cases}$$

$$\Downarrow$$

$$y = 1 - \frac{\alpha(1+\alpha)}{1+\alpha^2} = \frac{1-\alpha}{1+\alpha^2}$$

Dla $c_1 = 0 \rightarrow 1 + \alpha y = 0 \rightarrow y = -\frac{1}{\alpha}$

Dla $c_2 = 0 \rightarrow 1 - \alpha x = 0 \rightarrow x = \frac{1}{\alpha}$

Oczywiście $c_1 = c_2 = 0$. Zatem pkt. stacjonarne to $\left(\frac{1+\alpha}{1+\alpha^2}, \frac{1-\alpha}{1+\alpha^2}\right)$, $(0,0)$, $\left(\frac{1}{\alpha}, 0\right)$, $\left(0, -\frac{1}{\alpha}\right)$

$(\bar{x}, \bar{y})$

Nie interesuje nas

Niech $\varphi(t) = x(t) - \bar{x}$ Linearyzujemy. ?

$\psi(t) = y(t) - \bar{y}$.

$$\varphi' = f(\varphi + \bar{x}, \psi + \bar{y})$$

$$\psi' = g(\varphi + \bar{x}, \psi + \bar{y})$$ Ze wzoru Taylora

$$\begin{pmatrix} \varphi' \\ \psi' \end{pmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x}(\bar{x},\bar{y}) & \frac{\partial f}{\partial y}(\bar{x},\bar{y}) \\ \frac{\partial g}{\partial x}(\bar{x},\bar{y}) & \frac{\partial g}{\partial y}(\bar{x},\bar{y}) \end{bmatrix} + R(\varphi,\psi) = \begin{bmatrix} 1 - \bar{y} - 2\alpha\bar{x} & -\bar{x} \\ \bar{y} & -1 + \bar{x} - 2\alpha\bar{y} \end{bmatrix} + R(\varphi,\psi)$$

Ponad to $1-\bar{y}-2\alpha\bar{x} = 1-\frac{1-\alpha}{1+\alpha^2}-\frac{2\alpha+2\alpha^2}{1+\alpha^2} = 1-\frac{1+\alpha+2\alpha^2}{1+\alpha^2}$

$$= \frac{\cancel{1}+\alpha^2-\cancel{1}-\alpha-2\alpha^2}{1+\alpha^2} = -\alpha\frac{1+\alpha}{1+\alpha^2} = -\alpha\bar{x}$$

$$-1+\bar{x}-2\alpha\bar{y} = -1+\frac{1+\alpha}{1+\alpha^2}-2\alpha\frac{1-\alpha}{1+\alpha^2} =$$

$$= -1+\frac{1-\alpha+2\alpha^2}{1+\alpha^2} = \frac{-\alpha+\alpha^2}{1+\alpha^2} = \alpha\frac{\alpha-1}{1+\alpha^2} = -\alpha\bar{y}$$

$$\begin{vmatrix} -\alpha\bar{x}-\lambda & -\bar{x} \\ \bar{y} & -\alpha\bar{y}-\lambda \end{vmatrix} = (\alpha\bar{x}+\lambda)(\alpha\bar{y}+\lambda)+\bar{x}\bar{y} =$$

$$= \alpha^2\bar{x}\bar{y}+\lambda\alpha(\bar{x}+\bar{y})+\lambda^2+\bar{x}\bar{y} = 0$$

$$\Delta = \alpha^2(\bar{x}+\bar{y})^2-4(\alpha^2+1)\bar{x}\bar{y}$$

$$= \alpha^2(\bar{x}-\bar{y})^2-4\bar{x}\bar{y} =$$

~~Pytanie:~~ $= \alpha^2\left(\frac{1+\alpha-1+\alpha}{1+\alpha^2}\right)^2-4\frac{1-\alpha^2}{(1+\alpha^2)^2}$

$$= \alpha^2\frac{(2\alpha)^2}{(1+\alpha^2)^2}-4\frac{1-\alpha^2}{(1+\alpha^2)^2}$$

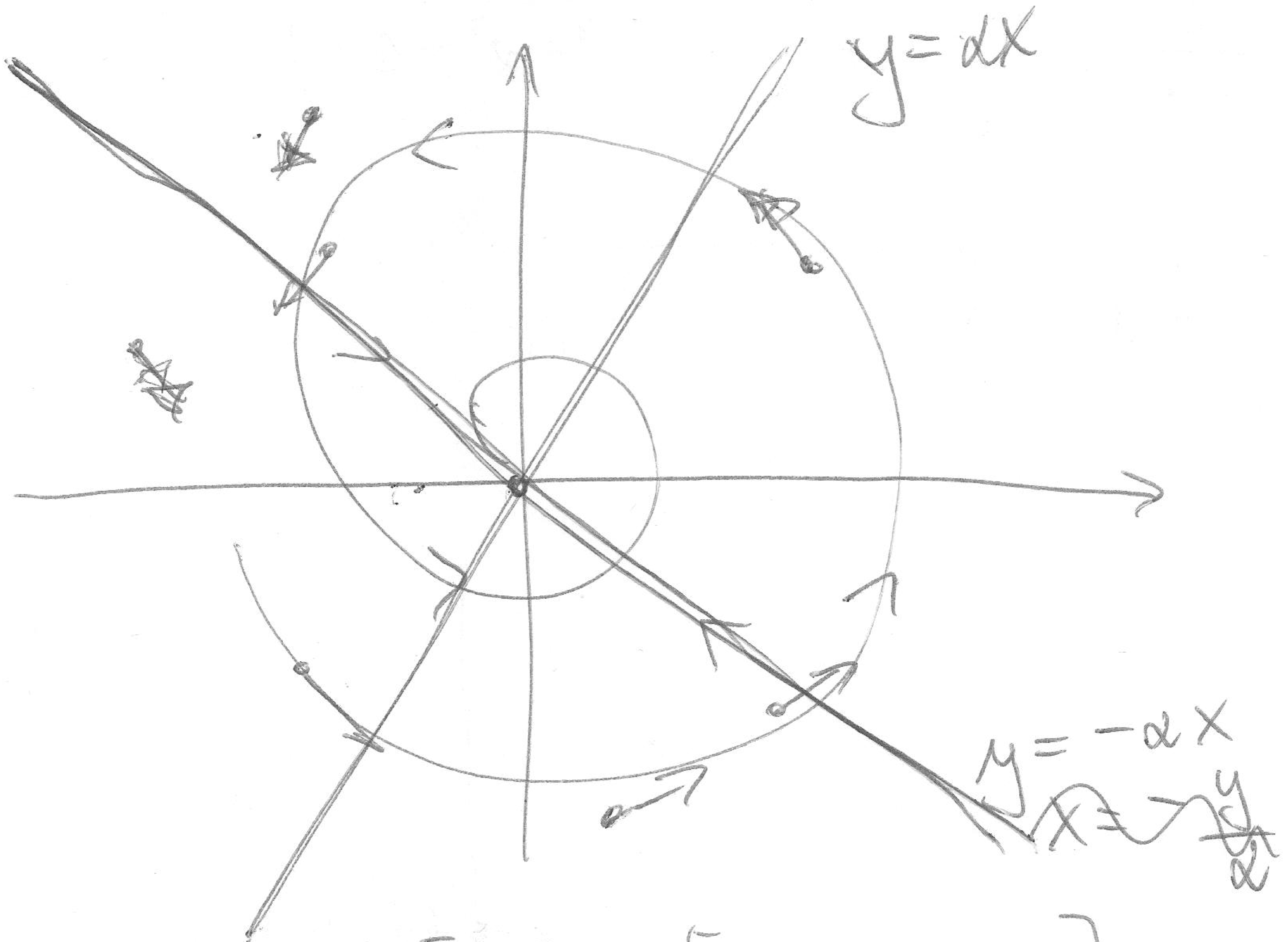
$$= \frac{4\alpha^4+4\alpha^2-4}{(1+\alpha^2)^2} = \frac{4(\alpha^4+\alpha^2-1)}{(1+\alpha^2)^2}$$

Pytanie: Czy $\mathrm{Re}\left(\frac{-\alpha(\bar{x}+\bar{y})\pm\frac{2}{1+\alpha^2}\sqrt{\alpha^4+\alpha^2-1}}{2}\right) \overset{?}{<} 0$

Wolfram tw. że to prawda dla $\alpha\in[0,1)$.

Dla $\alpha=0$ podstawowa wersja Lotki-Volterry.

Z tw. Grobmana-Hartmana portrety fazowe linearyzacji i wejściowego ~~zada~~ równania są homeomorficzne:



$$\begin{bmatrix} -\alpha\bar{x} & -\bar{x} \\ \bar{y} & -\alpha\bar{y} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\bar{x}(\alpha x+y) \\ \bar{y}(x-\alpha y) \end{bmatrix}$$