a)

$y' = 2$ Tylko funkcje liniowe mają pochodną stale równą liczbie niezerowej.

$y(x) = 2x + b$

$y(0) = b = 2$

$y(x) = 2x + 2$

b) $y' = \frac{y}{x}$. Przyjmujemy

$g(x) = \frac{1}{x}$, $f(y) = \frac{1}{y}$

Wtedy

$\frac{y'}{y} = \frac{1}{x}$

$\int \frac{y'(x)}{y(x)} dx = \int \frac{1}{x} dx$

$$y = y(x)$$

$$dy = \frac{dy}{dx}\, dx$$

$$\int \frac{1}{y}\, dy = \int \frac{1}{x}\, dx$$

$$\log y(x) = \log x + C$$

Ponendo $y(1) = 5$

$\log 5 = 0 + 5$, cioè $C = 5$.

$$\log y(x) = \log x + 5 \quad \Big|\, e^{\cdot}$$

$$y(x) = e^5 \cdot x$$

c)

$$y' = -y^2 e^x \quad , \quad f(y) = \frac{1}{-y^2}$$
$$g(x) = e^x$$

$$\frac{y'}{-y^2} = e^x$$

$$-\int \frac{y'}{y^2}\,dx = \int e^x\,dx$$

$$-\int \frac{1}{y^2}\,dy = \int e^x\,dx$$

$$\frac{1}{y} = e^x + C$$

Ponadto $y(0) = \frac{1}{2}$

$$2 = 1 + c$$

$$c = 1 \longrightarrow y(x) = \frac{1}{1+e^x}$$