Franciszek Malinka

Zad. 1 $\quad y'' - t^3 y = 0$

Zał. że $\quad y = \sum_{n=0}^{\infty} a_n t^n \quad$ dla pewnych a_n.

Wtedy $\quad y'' - t^3 y = \sum_{n=0}^{\infty} n(n-1) a_n t^{n-2} - t^3 \sum_{n=0}^{\infty} a_n t^n$

$$= \sum_{n=-2}^{\infty} (n+2)(n+1) a_{n+2} t^n - \sum_{n=0}^{\infty} a_n t^{n+3}$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} t^n - \sum_{n=0}^{\infty} a_n t^{n+3} =$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} t^n - \sum_{n=3}^{\infty} a_{n-3} t^n$$

$$= \sum_{n=3}^{\infty} (n+2)(n+1) a_{n+2} t^n - a_{n-3} t^n + 2a_2 + 6a_3 t + 12 a_4 t^2$$

$= 0$

Zatem $\cancel{2a_2 + 6a_3}$
$$\begin{cases} 2a_2 = 0 \\ 6a_3 = 0 \\ 12 a_4 = 0 \\ (n+2)(n+1) a_{n+2} - a_{n-3} = 0 \end{cases}$$

$$\Downarrow$$

$$a_{n+2} = \frac{a_{n-3}}{n^2 + 3n + 2} \qquad \boxed{a_{n+5} = \frac{a_n}{(n+5)(n+4)}}$$

Zatem a_0, a_1 determinują cały szereg.

$1 = y(0) = a_0 \quad , \quad 0 = y'(0) = a_1$.

Zatem $\quad y = \sum_{n=0}^{\infty} a_{5n} t^{5n}, \quad \cancel{a_{5n} = \frac{a_{5(n-1)}}{(5n+5)(5n+4)}}, \quad a_0 = 1$

$$\cancel{y = \sum_{n=0}^{\infty} 1 + \frac{t^5}{5 \cdot 4} + \frac{5 \cdot 4}{10 \cdot 9} t^{10} + \frac{10 \cdot 9}{5 \cdot 4 \cdot 5 \cdot 4} \cdot t^{15}}$$