Franciszek Malinka

Zad 1 $\quad y'' - t^3 y = 0$

Zał. że $\quad y = \sum_{n=0}^{\infty} a_n t^n \quad$ dla pewnych a_n.

Wtedy $\quad y'' - t^3 y = \sum_{n=0}^{\infty} n(n-1) a_n t^{n-2} - t^3 \sum_{n=0}^{\infty} a_n t^n$

$$= \sum_{n=-2}^{\infty} (n+2)(n+1) a_{n+2} t^n - \sum_{n=0}^{\infty} a_n t^{n+3}$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} t^n - \sum_{n=0}^{\infty} a_n t^{n+3} =$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} t^n - \sum_{n=3}^{\infty} a_{n-3} t^n$$

$$= \sum_{n=3}^{\infty} (n+2)(n+1) a_{n+2} t^n - a_{n-3} t^n + 2a_2\cancel{t} + 6a_3 t + 12 a_4 t^2$$

$$= 0$$

Zatem $\cancel{2a_2 + 6a_3}$ $\begin{cases} 2a_2 = 0 \\ 6a_3 = 0 \\ 12 a_4 = 0 \\ (n+2)(n+1) a_{n+2} - a_{n-3} = 0 \end{cases}$

$$\Downarrow$$

$$a_{n+2} = \frac{a_{n-3}}{n^2 + 3n + 2} \qquad \boxed{a_{n+5} = \frac{a_n}{(n+5)(n+4)}}$$

Zatem a_0, a_1 determinują cały szereg.

$1 = y(0) = a_0$, $\quad 0 = y'(0) = a_1$.

Zatem $\quad y = \sum_{n=0}^{\infty} a_{5n} t^{5n}$, $\quad \cancel{a_{5n} = \frac{a_{5(n-1)}}{(5n+5)(5n+4)}}$, $\quad a_0 = 1$

$$\cancel{y = \sum_{n=0}^{\infty} 1 + \frac{t^5}{5\cdot 4} + \frac{5\cdot 4}{10\cdot 9} t^{10} + \frac{10\cdot 9}{5\cdot 4\cdot 5\cdot 4} t^{15}}$$

Zad. 2

$y'' + y = t^2 \cos t$, Niech $\mathcal{L}\{y\}(s) = F(s)$, wtedy

$$s^2 F(s) - s\,\overset{1}{\overset{\|}{y(0)}} - \overset{1}{\overset{\|}{y'(0)}} + F(s) = \mathcal{L}\{t^2 \cos(t)\}(s)$$

$$\|$$

$$\frac{d^2}{ds^2}\mathcal{L}\{\cos t\}$$

$$\|$$

$$\frac{d^2}{ds^2}\frac{s}{s^2+1} \quad \text{(TABELKA)}$$

$$\|$$

$$\frac{2s(s^2-3)}{(1+s^2)^3}$$

$$F(s)(s^2+1) = \frac{2s(s^2-3)}{(1+s^2)^3} + s + 1$$

$$F(s) = \frac{2s(s^2-3)}{(1+s^2)^4} + \frac{1+s}{1+s^2}$$

$$y = \mathcal{L}^{-1}\{F\} = \mathcal{L}^{-1}\left\{\frac{2s(s^2-3)}{(1+s^2)^4}\right\} + \mathcal{L}^{-1}\left\{\frac{1+s}{1+s^2}\right\}$$

$$\| \qquad\qquad \|$$

$$\mathcal{L}^{-1}\left\{\frac{2s^3}{(1+s^2)^4} - \frac{6s}{(1+s^2)^4}\right\} \qquad \cos t + \sin t$$

$\|$ WOLFRAM

$$\frac{1}{12}\left(2t^3 \sin t + 3t^2 \cos t - 3t \sin t\right)$$

$$y(t) = \frac{1}{12}\left(2t^3 \sin t + 3t^2 \cos t - 3t \sin t\right) + \cos t + \sin t$$

Zad. 3

$$x' = 2x + y + xy$$
$$y' = x + y + x^2 y$$

Możemy zlinearyzować, bo $\frac{xy}{\left\|\begin{bmatrix}x\\y\end{bmatrix}\right\|} \xrightarrow{\begin{bmatrix}x\\y\end{bmatrix}\to 0} 0$

$\frac{x^2 y}{\left\|\begin{bmatrix}x\\y\end{bmatrix}\right\|} \xrightarrow{\begin{bmatrix}x\\y\end{bmatrix}\to 0} 0$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = (2-\lambda)(1-\lambda) - 1 = \lambda^2 - 3\lambda + 1$$

$$\lambda_1 = \frac{-\sqrt{5}+3}{2}, \quad \lambda_2 = \frac{\sqrt{5}+3}{2}$$

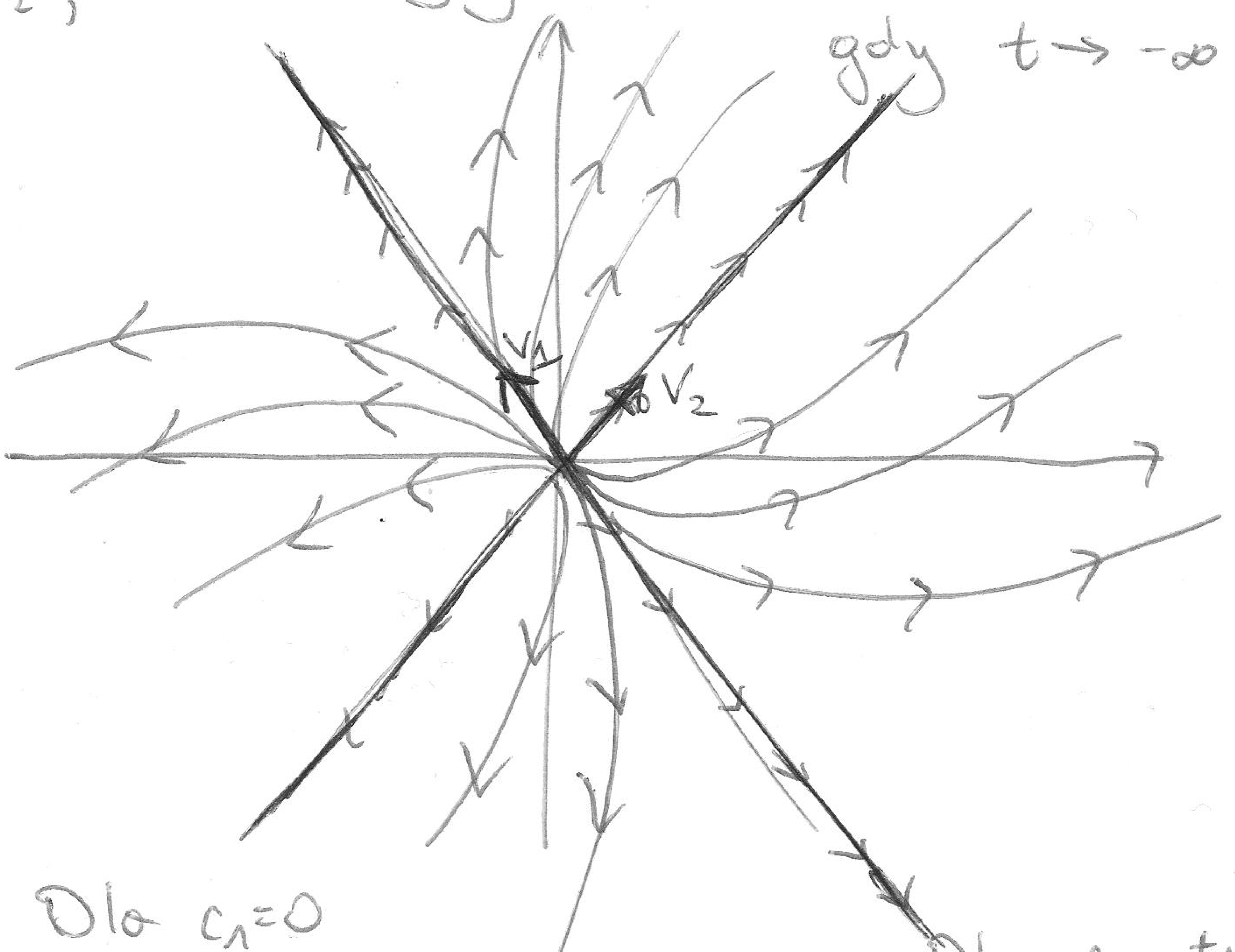
Zatem równanie niestabilne, bo $\lambda_2 > 0$.

Wektory własne to $v_1 = \begin{bmatrix} \frac{-\sqrt{5}+1}{2} \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} \frac{\sqrt{5}+1}{2} \\ 1 \end{bmatrix}$

rozwiązania $x(t) = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2$ (Wolfram)

$0 < \lambda_1 < \lambda_2$, ~~blisko~~ 0 gdy $t \to \infty$ to wektor v_2 dominuje

gdy $t \to -\infty$ to v_1 dominuje, stąd blisko 0 wektory są „prawie styczne" do wektora v_1, gdy $t \to \infty$ są „prawie styczne" do v_2.



Dla $c_1 = 0$, $c_2 = \pm 1$

Dla $c_1 = \pm 1$, $c_2 = 0$