Zad. 2

$$X_i \sim Exp(1) \iff f_{X_i}(x) = e^{-x} \mathbb{1}_{[0,\infty)}, \quad \mathbb{E}X_i = 1$$

$$\frac{X_1 + X_2 + \dots + X_n + n}{n} = \frac{X_1 + \dots + X_n}{n} + 1 \xrightarrow[p.w.]{MPWL} \mathbb{E}X_1 + 1 = 2$$

$$\frac{X_1^2 + \dots + X_n^2 + \sqrt{n}}{n} = \frac{X_1^2 + \dots + X_n^2}{n} + \frac{1}{\sqrt{n}} = (*)$$

$$\mathbb{E}X_1^2 = \int_{\mathbb{R}} x^2 f_{X_1}(x)\,dx = \int_0^\infty x^2 e^{-x}\,dx =$$

$$= -x^2 e^{-x}\Big|_0^\infty + \int_0^\infty 2x e^{-x}\,dx$$

$$= \underbrace{-x^2 e^{-x}\Big|_0^\infty}_{=0} + 2\int_0^\infty x e^{-x}\,dx$$

$$= 0 + 2\mathbb{E}X_1 = 2$$

$$(*) \xrightarrow[p.w.]{MPWL} 2$$

Stąd

$$\frac{X_1 + \dots + X_n + n}{X_1^2 + \dots + X_n^2 + \sqrt{n}} = \frac{\frac{X_1 + \dots + X_n + n}{n}}{\frac{X_1^2 + \dots + X_n^2 + \sqrt{n}}{n}} \longrightarrow 1$$