$$= \int_{\mathbb{R}} \tfrac{1}{3} e^{-\frac{(x-y)}{3}} \cdot \mathbb{1}_{[0,\infty)}(x-y) \cdot e^{y} \mathbb{1}_{(-\infty,0]}(y) \, dy$$

$$= \int_{-\infty}^{0} \tfrac{1}{3} e^{-\frac{(x-y)}{3}} \cdot e^{y} \cdot \mathbb{1}_{[0,\infty)}(x-y) \, dy =$$

$$= \int_{-\infty}^{\min\{x,0\}} \tfrac{1}{3} e^{-\frac{(x-y)}{3}} e^{y} \, dy = \int_{-\infty}^{\min\{x,0\}} \tfrac{1}{3} e^{-\frac{x}{3}+\frac{y}{3}} e^{y} \, dy =$$

$$= \int_{-\infty}^{\min\{x,0\}} \tfrac{1}{3} e^{-\frac{x}{3}} \cdot e^{\frac{y}{3}} \cdot e^{y} \, dy = \tfrac{1}{3} e^{-\frac{x}{3}} \int_{-\infty}^{\min\{x,0\}} e^{\frac{4y}{3}} \, dy$$

$$= \tfrac{1}{3} e^{-x/3} \cdot \left(\tfrac{3}{4} e^{4/3 y} \Big|_{-\infty}^{\min\{x,0\}} \right) =$$

$$\cancel{= \tfrac{1}{3} e^{-x/3} \cdot \left(3/4 \, e^{4/3 x} - 0 \right) = \tfrac{1}{4} e^{x}}$$

$$= \begin{cases} \cancel{\tfrac{1}{3} e^{-x/3}} \; 1/4 \, e^{x} & \text{dla } x < 0 \\ \tfrac{1}{4} e^{-x/3} & \text{dla } x \geq 0 \end{cases} \quad \text{(ciągłe w 0!)}$$

Zatem $\mathbb{E} Z_k = \mathbb{P}[0 > 3X_{2k-1} - X_{2k}] = \int_{-\infty}^{0} \cancel{\tfrac{1}{4} e^{-x/3}}$

$$\cancel{= \tfrac{1}{4} \cdot (-3) e^{-\frac{x}{3}} \Big|_{-\infty}^{0}} = \int_{-\infty}^{0} 1/4 \, e^{x} \, dx = 1/4 \, e^{x} \Big|_{-\infty}^{0} = \tfrac{1}{4}$$

Zatem z MPWL mamy (Zał. są spełnione, $\mathbb{E} Z_k$ istn. oraz $\{Z_k\}$ i.i.d.)

$$\frac{Z_1 + \ldots + Z_n}{n} \xrightarrow[n \to \infty]{} 1/4$$