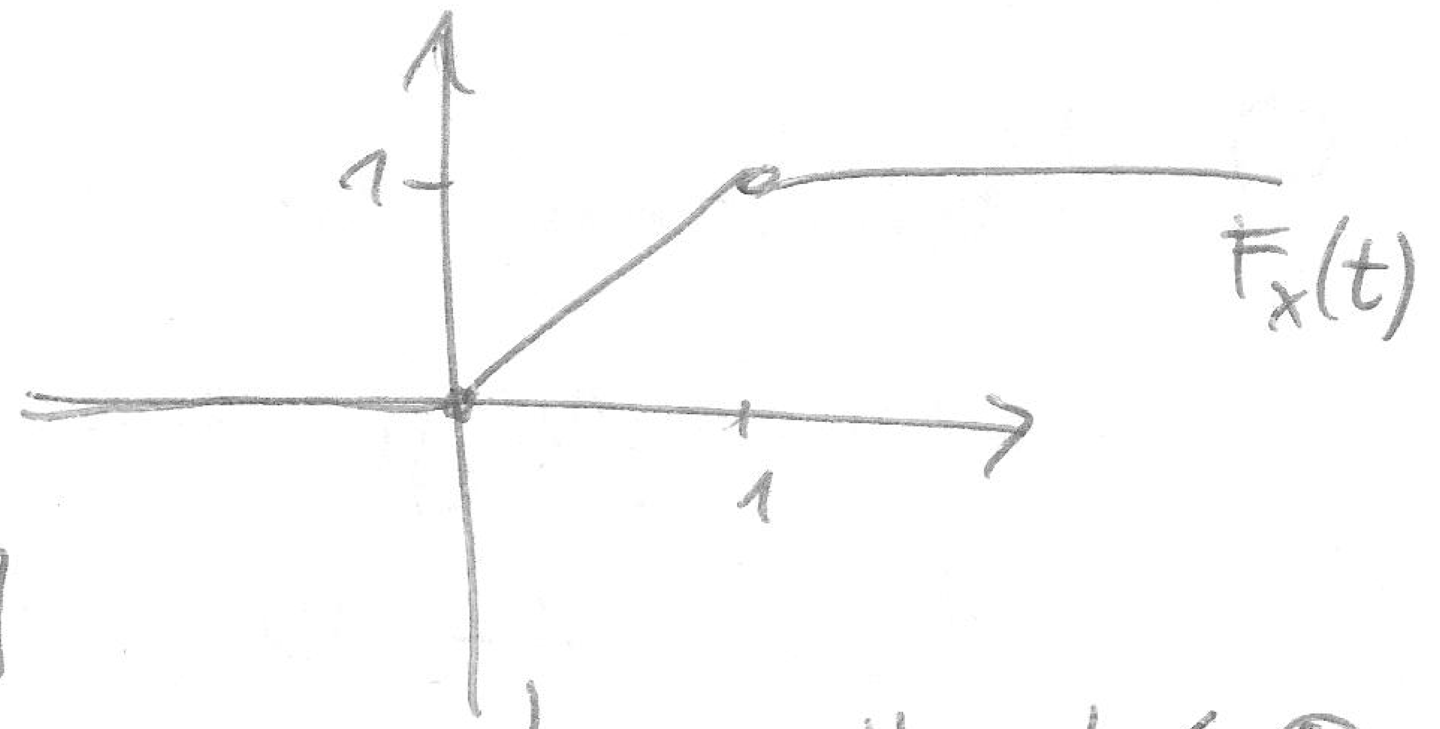
Zad. 2 $\{X_n\}$ i.i.d., $X_1 \sim \mathcal{U}([0,1])$



a) ~~Tarpaw~~ Policzmy dystrybuantę zmiennej $Y_1 = 1 - X_1$: $F_{Y_1}(t) = \mathbb{P}[Y_1 \leq t]$

$$= \mathbb{P}[\cancel{X}1 - X_1 \leq t] = \mathbb{P}[1 - t \leq X_1] = \begin{cases} 0 & \text{dla } t \leq 0 \\ \cancel{t} & \text{dla } t < 1 \\ 1 & \text{dla } t \geq 1 \end{cases}$$

Zatem $F_{Y_1} = F_{X_1}$, czyli $1 - X_1$ i X_1 mają ten sam rozkład.

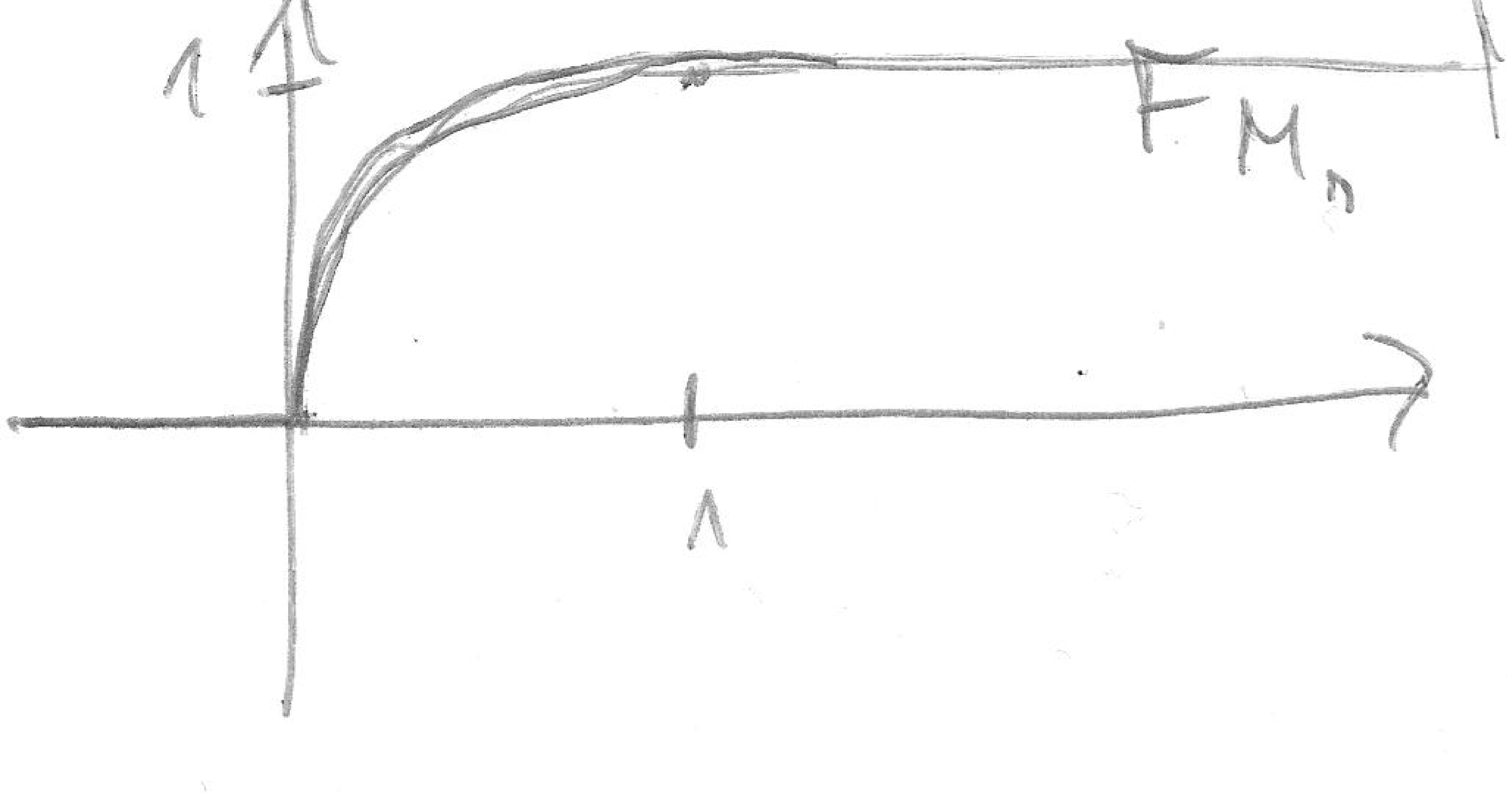
b) $M_n = \min\limits_{1 \leq k \leq n} X_k$. ~~Weźmy $\varepsilon > 0$. Chcemy pokazać, że $\mathbb{P}[|M_n| \geq \varepsilon] \xrightarrow[n\to\infty]{} 0$.~~

~~Chcemy pokazać, że $M_n \xrightarrow{d} 0$. To implikuje $M_n \xrightarrow{\mathbb{P}} 0$.~~ Rozkład M_n:

$$\mathbb{P}[M_n \leq t] = \mathbb{P}[X_1 \leq t \text{ lub } X_2 \leq t \text{ lub} \ldots \text{ lub } X_n \leq t] =$$

$$= 1 - \mathbb{P}[X_1 \geq t \text{ i } \ldots \text{ i } X_n \geq t] \overset{\text{nzl.}}{=} 1 - \mathbb{P}[X_1 \geq t] \ldots \mathbb{P}[X_n \geq t]$$

$$= 1 - \prod_{k=1}^{n} (1 - \mathbb{P}[X_k \leq t]) = \begin{cases} 1 - (1-t)^n & \text{dla } t \in [0,1] \\ \cancel{1} - 1 = 0 & \text{dla } t < 0 \\ 1 - 0 = 1 & \text{dla } t \geq 1 \end{cases}$$



Dla $\varepsilon \in [0,1]$

$$\mathbb{P}[M_n \geq \varepsilon] =$$
$$= 1 - \mathbb{P}[M_n \leq \varepsilon]$$
$$= 1 - (1 - (\underset{\substack{\wedge \\ 1}}{1-\varepsilon})^n) \xrightarrow{n\to\infty} 0$$

Zatem $M_n \xrightarrow{\mathbb{P}} 0$.