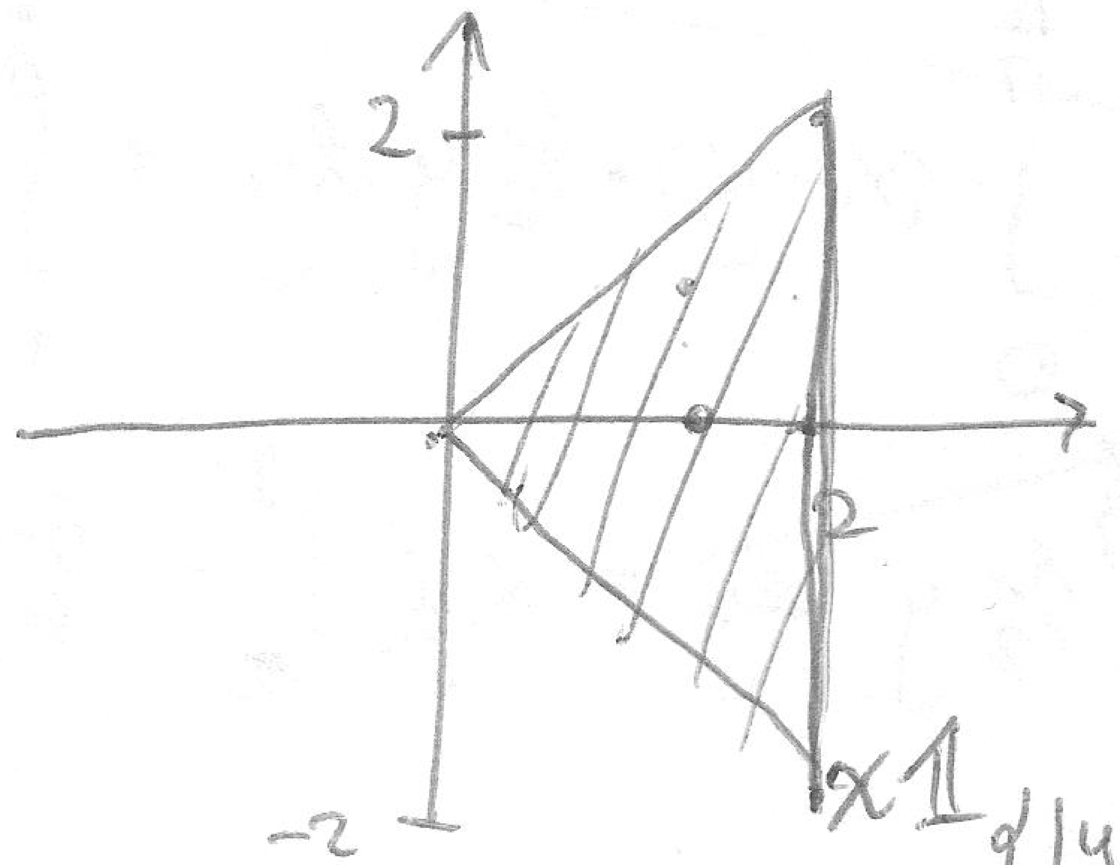
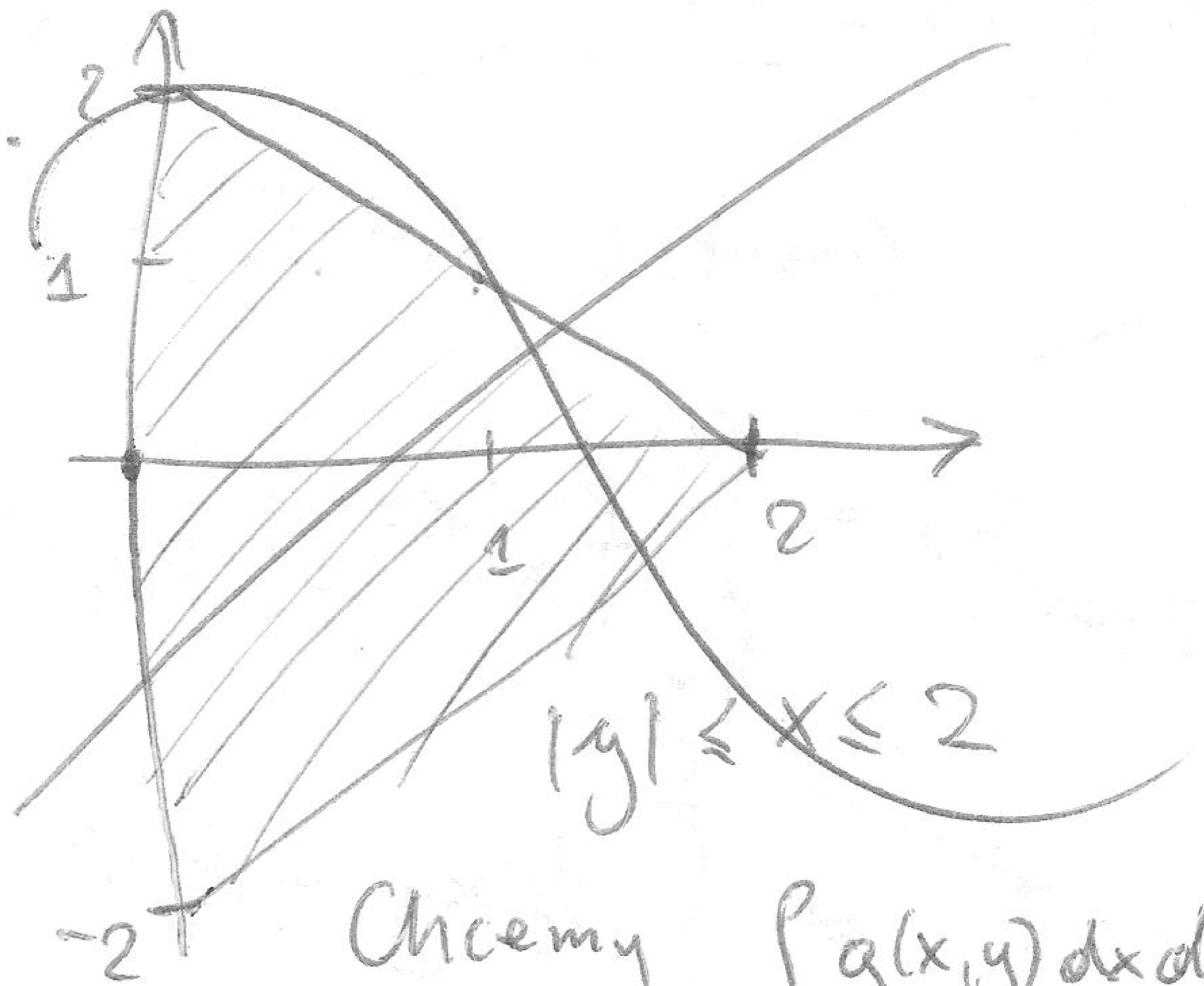
Zad. 1

a) Chcemy, żeby $\int_{\mathbb{R}^2} g(x,y)\,dx\,dy = 1$.



$x\mathbb{1}_{\{|y|\le x\le 2\}}$



Chcemy $\int_{\mathbb{R}^2} g(x,y)\,dx\,dy = 1$

Czyli $\int_0^2\int_{-x}^{x} Cx\,dy\,dx = 1$

b) $\mu_X(A) = \mathbb{P}[X\in A, Y\in\mathbb{R}] = \int_{A\times\mathbb{R}} g(x,y)\,dx\,dy \overset{\text{Tw. Fubiniego}}{=} \int_A\int_{\mathbb{R}} g(x,y)\,dy\,dx$

$A \in \mathrm{Bor}([0,2])$

$= \int_A\int_{-x}^{x} g(x,y)\,dy\,dx =$

KONTYNUACJA a)

$\int_0^2\int_{-x}^{x} x\,dy\,dx = 1/C$

$\int_0^2 yx\Big|_{-x}^{x} dx = \int_0^2 2x^2\,dx = \frac{2}{3}x^3\Big|_0^2 = \frac{16}{3}$

Czyli $C = 3/16$

$= \int_A\int_{-x}^{x} \frac{3}{16}x\,\mathbb{1}_{\{|y|\le x\le 2\}}\,dy\,dx =$

$= \int_A \frac{3}{16}yx\Big|_{-x}^{x} dx = \int_A \frac{3}{8}x^2\,dx$, czyli $f_X(x) = \frac{3}{8}x^2\cdot\mathbb{1}_{[0,2]}$

(oczywiście myślimy, że f_X określone na $[0,2]$)

$\mu_Y(A) = \mathbb{P}[X\in\mathbb{R}, Y\in A] =$

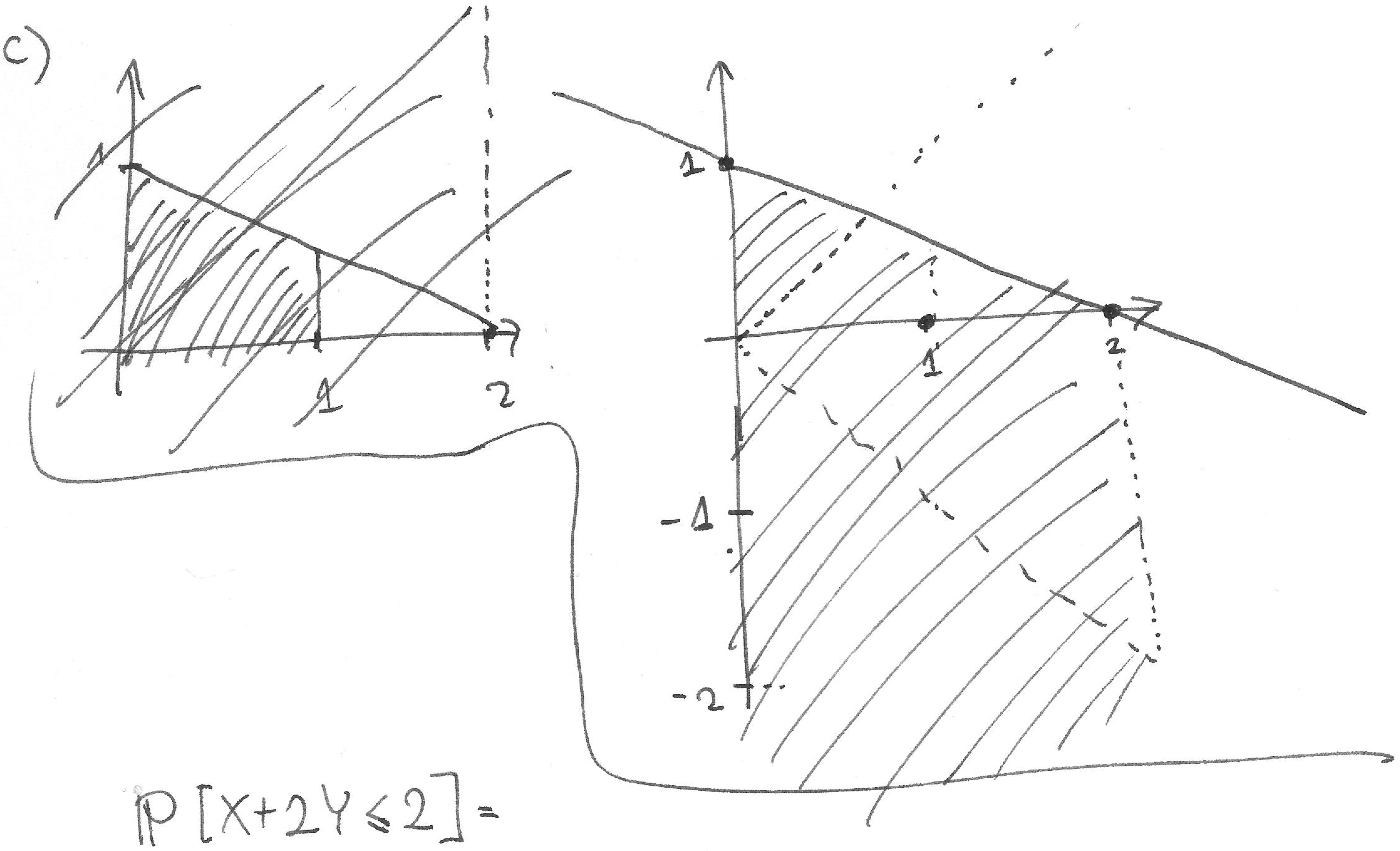
$A \subset \mathrm{Bor}([-2,2])$

$\overset{\text{Fubini}}{=} \int_A\int_{|y|}^{2} \frac{3}{16}x\,dx\,dy = \int_A \frac{3}{32}x^2\Big|_{|y|}^{2} dy = \int_A \frac{3}{8} - \frac{3}{32}|y|^2\,dy$

Czyli $f_Y(y) = \left[\frac{3}{8} - \frac{3}{32}y^2\right]\cdot\mathbb{1}_{[-2,2]}$

(Spr. $\int_0^2 \frac{3}{8}x^2\,dx = \frac{1}{8}x^3\Big|_0^2 = 1$, $\int_{-2}^{2} \frac{3}{8} - \frac{3}{32}y^2\,dy = \frac{3}{8}y - \frac{1}{32}y^3\Big|_{-2}^{2} = \frac{3}{4} - \frac{1}{4} + \frac{3}{4} - \frac{1}{4} \underset{\text{GIT}}{=} 1$)

c)



$\mathbb{P}[X+2Y \leq 2] =$

$= \cancel{\int_0^2 \int_{-2}^{\frac{x-2}{2}}}$

d) $f_X(x) = \frac{3}{8}x^2 \cdot \mathbb{1}_{[0,2]}(x)$, $f_Y(y) = \left[\frac{3}{8} - \frac{3}{32}y^2\right]\mathbb{1}_{[-2,2]}(y)$

~~$\mathbb{P}[X, Y \in$~~

X, Y są niezal. $\iff f_X(x)\cdot f_Y(y) = g(x,y)$,

ale tak nie jest:

$\frac{3}{8}x^2 \mathbb{1}_{[0,2]}(x) \cdot \left[\frac{3}{8} - \frac{3}{32}y^2\right]\mathbb{1}_{[-2,2]}(y) =$

$= \mathbb{1}_{\{|y| \leq x \leq 2\}} \cdot \left[\frac{9}{64}x^2 - \frac{9}{256}x^2y^2\right] \neq x \cdot \mathbb{1}_{\{\dots\}}(x)$