zad. 3

$$0 = f(\alpha) = f'(\alpha) \neq f''(\alpha) \qquad \Phi(x) = x - \frac{f(x)}{f'(x)}$$

$$\lim_{x\to\alpha} \Phi(x) = \lim_{x\to\alpha}\left(x - \frac{f(x)}{f'(x)}\right) \overset{H[\frac{0}{0}]}{=} \alpha - \lim_{x\to\alpha}\frac{f'(x)}{f''(x)} = \alpha$$

($f'(x) \to 0$, $f''(x) \neq 0$)

$\alpha \notin D(\Phi)$ ale możemy uciąglić Φ i powiedzieć $\Phi(\alpha) = \alpha$.

$$\lim_{x\to\alpha} \Phi'(x) = \lim_{x\to\alpha} 1 - \frac{[f'(x)]^2 - f(x)f''(x)}{[f'(x)]^2} = \lim_{x\to\alpha}\left(1 - 1 + \frac{f(x)f''(x)}{[f'(x)]^2}\right) =$$

$$\overset{H[\frac{0}{0}]}{=} \lim_{x\to\alpha} \frac{f'(x)f''(x) + f(x)f'''(x)}{2f'(x)\cdot f''(x)} = \lim_{x\to\alpha}\left(\frac{1}{2} + \frac{f(x)f'''(x)}{2f'(x)f''(x)}\right) =$$

$$\overset{H[\frac{0}{0}]}{=} \lim_{x\to\alpha}\left(\frac{1}{2} + \frac{f'(x)f'''(x) + f(x)f^{IV}(x)}{2[f''(x)]^2 + 2f'(x)f'''(x)}\right) = \frac{1}{2}$$

Stąd α $|\Phi'(x)| < 1$, więc Φ zbieżna liniowo dla x bliskich α.