zad. 12 $\quad f(x,y) = e^{\sin x} \cdot \log(1+x+y)$

Niech $g(x) = e^{\sin x}$, $h(x,y) = \log(1+x+y)$. $\quad g'(x) = e^{\sin x}\cos x$, $\quad g''(x) = \underbrace{e^{\sin x}}_{1}\underbrace{\cos^2 x}_{1} + -e^{\sin x}\underbrace{\sin x}_{0}$

$$g(x) = g(0) + \frac{dg}{dx}(0)x + \frac{1}{2}\frac{d^2g}{dx^2}(0)x^2 + R_g(x) = 1 + x + \frac{x^2}{2} + R_g(x)$$

$$h(x,y) = h(0,0) + \frac{\partial h}{\partial x}(0,0)x + \frac{\partial h}{\partial y}(0,0)y + \frac{1}{2}\frac{\partial^2 h}{\partial x^2}(0,0)x^2 + \frac{\partial^2 h}{\partial x \partial y}(0,0)xy + \frac{1}{2}\frac{\partial^2 h}{\partial y^2}(0,0)y^2 + R_h(x,y)$$

$$= 1 + x + y - \frac{1}{2}x^2 - xy - \frac{1}{2}y^2 + R_h(x,y)$$

One są sobie równe! $\quad -\frac{1}{(1+x+y)^2}$

$$f(x,y) = g(x)\cdot h(x,y) = \left(1 + x + \frac{x^2}{2} + R_g(x)\right)\cdot\left(1 + x + y - \underbrace{\frac{1}{2}(x+y)^2}_{\frac{1}{2}x^2 + xy + \frac{1}{2}y^2} + R_h(x,y)\right)$$

$$= 1 + x + y - \frac{1}{2}x^2 - xy - \frac{1}{2}y^2 + x + x^2 + xy - \frac{1}{2}x^3 - x^2y - \frac{1}{2}xy^2 +$$
$$+ \frac{x^2}{2} + \frac{x^3}{2} + \frac{x^2y}{2} - \frac{1}{4}x^4 - \frac{1}{2}x^3y - \frac{1}{4}x^2y^2 +$$
$$+ R_g(x)h(x,y) + R_h(x,y)g(x) =$$

$$= 1 + 2x + y + x^2 - \frac{1}{2}y^2 + \frac{1}{2}x^2y - \frac{1}{2}xy^2 + \frac{1}{4}x^4 - \frac{1}{2}x^3y - \frac{1}{4}x^2y^2 +$$
$$R_g(x)h(x,y) + R_h(x,y)g(x)$$

$$\lim_{(x,y)\to(0,0)} \frac{R_g(x)h(x,y) + R_h(x,y)g(x)}{\|(x,y)\|^2} = \lim_{(x,y)\to 0} \underbrace{\frac{R_g(x)}{\|x\|^2}}_{\to 0}\underbrace{h(x,y)}_{\to 1} + \underbrace{\frac{R_h(x,y)}{\|(x,y)\|^2}}_{\to 0}\underbrace{g(x)}_{\to 1}$$

$= 0$. Stąd i zad. 8 mamy to co chcieliśmy.