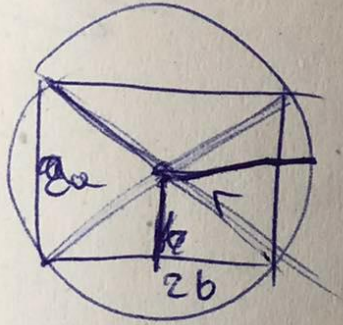


$n=0$

(\*)

zad. 24\*

$$a, b, c > 0$$



$$\begin{cases} f(a, b) = 4ab \\ g(a, b) = b^2 + a^2 = r^2 \end{cases}$$

$$\nabla f(a, b) = (4b, 4a) = \lambda (2a, 2b)$$

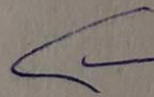
$\Downarrow$   
 ~~$4b = 2a$~~

$$\begin{cases} 4b = \lambda 2a \\ 4a = \lambda 2b \\ b^2 + a^2 = r^2 \end{cases} \Rightarrow 4a = \lambda^2 a \Rightarrow \lambda = 2$$

$\Downarrow$   
 $a = b = \frac{\sqrt{2}r}{2}$

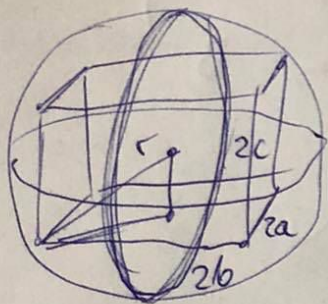
Maks. wartość  
 $f$  jest w  
punkcie

$$\left( \frac{\sqrt{2}r}{2}, \frac{\sqrt{2}r}{2} \right) \xrightarrow{f} 2r^2$$





$$f(a, b, c) = 8abc \quad g(a, b, c) = a^2 + b^2 + c^2 = r^2$$



$$\nabla f(a, b, c) = (8bc, 8ac, 8ab) =$$

$$= \lambda \nabla g(a, b, c) = \lambda (2a, 2b, 2c)$$

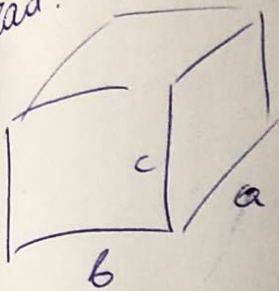
$$\left\{ \begin{array}{l} 8bc = \lambda 2a \\ 8ac = \lambda 2b \\ 8ab = \lambda 2c \\ a^2 + b^2 + c^2 = r^2 \end{array} \right. \rightarrow \begin{array}{l} 16abc^2 = \lambda^2 ab \Rightarrow c = \frac{\lambda}{4} \\ 16a^2bc = \lambda^2 bc \Rightarrow a = \frac{\lambda}{4} \\ 16ab^2c = \lambda^2 ac \Rightarrow b = \frac{\lambda}{4} \end{array}$$

$$3a^2 = r^2 \Rightarrow a = \frac{\sqrt{3}r}{3} = \frac{\lambda}{4} \Rightarrow \lambda = \frac{4\sqrt{3}r}{3}$$

Funkcja  $f$  przyjmuje swoje maksimum warunkowe w punkcie

$$\left( \frac{\sqrt{3}r}{3}, \frac{\sqrt{3}r}{3}, \frac{\sqrt{3}r}{3} \right) \xrightarrow{f} 8 \cdot \frac{3\sqrt{3}r^3}{27} = \frac{8\sqrt{3}r^3}{9}$$

rad. 18



$$a, b, c > 0$$

$$f(a, b, c) = abc$$

$$g(a, b, c) = a + 2b + 2c = 108$$

$$\nabla f \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} bc \\ ac \\ ab \end{pmatrix} = \lambda \nabla g \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$\left\{ \begin{array}{l} bc = \lambda \\ ac = 2\lambda \\ ab = 2\lambda \\ a + 2b + 2c = 108 \end{array} \right. \begin{array}{l} \leadsto \\ \leadsto \\ \leadsto \\ \checkmark \end{array} \begin{array}{l} ac = 2bc \Rightarrow a = 2b \\ b = c \end{array}$$

$$2b + 2b + 2b = 108$$

$$a = 36$$

$$b = 18$$

$$c = 18$$

$f$  przyjmuje maks. wartość w punkcie  $\begin{pmatrix} 36 \\ 18 \\ 18 \end{pmatrix} \xrightarrow{f} 11664$