$$F_X(t) = \int_{-\infty}^{t} \frac{1}{\pi}\frac{1}{1+x^2}\,dx = \frac{1}{\pi}\left(\operatorname{arctg}(t) + \frac{\pi}{2}\right) =$$

$$= \frac{\operatorname{arctg}(t)}{\pi} + \frac{1}{2}$$

1° $t \geq 0$

$$F_{\frac{1}{X}}(t) = \mathbb{P}\left[\frac{1}{X} \leq t\right] = \mathbb{P}\left[X \geq 0 \text{ i } \frac{1}{X} \leq t\right] +$$

$$+ \mathbb{P}\left[X < 0 \text{ i } \frac{1}{X} \leq t\right] =$$

$$= \mathbb{P}\left[X \geq \frac{1}{t}\right] + \mathbb{P}[X < 0] = 1 - \mathbb{P}\left[X < \frac{1}{t}\right]$$

$$+ \mathbb{P}[X < 0] = 1 - F_X\left(\frac{1}{t}\right) + F_X(0) =$$

$$= 1 - \frac{\operatorname{arctg}\frac{1}{t}}{\pi} - \frac{1}{2} + \frac{1}{2} = -\frac{\operatorname{arctg}\frac{1}{t}}{\pi} + 1$$

2° $t < 0$

$$F_{\frac{1}{X}}(t) = \mathbb{P}\left[\frac{1}{X} \leq t\right] = \mathbb{P}\left[\frac{1}{X} \leq t \text{ i } X \geq 0\right] +$$

$$+ \mathbb{P}\left[\frac{1}{X} \leq t \text{ i } X < 0\right] =$$

$$= \mathbb{P}[\emptyset] + \mathbb{P}\left[0 > X \geq \frac{1}{t}\right] =$$

$$= F_X(0) - F_X\left(\frac{1}{t}\right) = \frac{\operatorname{arctg} 0}{\pi} + \frac{1}{2} -$$

$$\frac{\operatorname{arctg}\frac{1}{t}}{\pi} - \frac{1}{2} = -\frac{\operatorname{arctg}\frac{1}{t}}{\pi}$$

$$F_{\frac{1}{X}}(t) = \begin{cases} -\frac{\operatorname{arctg}\left(\frac{1}{t}\right)}{\pi} + 1 & \text{dla } t \geq 0 \\ -\frac{\operatorname{arctg}\left(\frac{1}{t}\right)}{\pi} & \text{dla } t < 0 \end{cases}$$

$$F_{\frac{1}{X}}'(t) = \begin{cases} -\frac{1}{\pi} \cdot \frac{1}{1+\frac{1}{t^2}} \cdot \left(-\frac{1}{t^2}\right) & \text{dla } t \geq 0 \\ -\frac{1}{\pi} \frac{1}{1+\frac{1}{t^2}} \cdot \left(-\frac{1}{t^2}\right) & \text{dla } t < 0 \end{cases}$$

$$F_{\frac{1}{X}}'(t) = \frac{1}{\pi} \cdot \frac{1}{1+t^2} = f_{\frac{1}{X}}(t)$$

(Dla 0 w sumie nie wiadomo, ale $\mu(\{0\}) = 0$)

Skoro gęstości te same, to rozkłady też.