Zad. 8 $\quad x_{n+1} = \dfrac{x_n(x_n^2+3R)}{3x_n^2+R} \xrightarrow[\text{zbiega sześciennie.}]{} \sqrt{R}$

$$\Phi(x) = \frac{x(x^2+3R)}{3x^2+R},$$ musi spełniać warunki

$$\Phi(\sqrt{R}) = \sqrt{R}, \quad \Phi'(\sqrt{R}) = \Phi''(\sqrt{R}) = 0, \; 0 < |\Phi'''(\sqrt{R})| < 1$$

spełniony, bo

$$\Phi(\sqrt{R}) = \frac{\sqrt{R}(R+3R)}{4R} = \sqrt{R}$$

- $$\Phi'(x) = \frac{[(x^2+3R)+x(2x)]\cdot(3x^2+R) - x(x^2+3R)(6x)}{(3x^2+R)^2} =$$

$$= \frac{(3x^2+3R)(3x^2+R) - 6x^2(x^2+3R)}{(3x^2+R)^2}$$

$$= \frac{9x^4+12x^2R+3R^2 - 6x^4 - 18x^2R}{(3x^2+R)^2} =$$

$$= \frac{3x^4-6x^2R+3R^2}{(3x^2+R)^2} = \frac{3(x^2-R)^2}{(3x^2+R)^2}$$

$$\Phi'(\sqrt{R}) = \frac{3\cdot(R-R)^2}{\text{coś}} = 0$$

- $$\Phi''(x) = \frac{6(x^2-R)\cdot 2x\cdot(3x^2+R)^2 - 3(x^2-R)^2\, 2(3x^2+R)\cdot 6x}{(3x^2+R)^4} =$$

$$= \frac{12x(x^2-R)(3x^2+R)[(3x^2+R) - 3(x^2-R)]}{(3x^2+R)^4} =$$

$$= \frac{12x(x^2-R)\cdot 4R}{(3x^2+R)^3} = \frac{48xR(x^2-R)}{(3x^2+R)^3}$$

$$\Phi''(\sqrt{R}) = \frac{48\sqrt{R}R(\overset{0}{R-R})}{\text{coś}} = 0$$

- $$\Phi'''(x) = \frac{[48R[(x^2-R)+2x^2)]](3x^2+R)^3 - 48xR(x^2-R)3(3x^2+R)^2 6x}{(3x^2+R)^6} =$$

$$(3x^2+R)^6$$

$$= \frac{48R(3x^2-R)(3x^2+R)^3 - 48R6x^2(x^2-R)\cdot 3(3x^2+R)^2}{(3x^2+R)^6} =$$

$$= \frac{48R\left[(3x^2-R)(3x^2+R) - 18x^2(x^2-R)\right]}{(3x^2+R)^4} = \frac{48R\left[9x^4-R^2-18x^4+18x^2R\right]}{(3x^2+R)^4} =$$

$$= \frac{48R(-9x^4+18x^2R-R^2)}{(3x^2+R)^4}$$

$\neq 0$ tylko dla $R=0$ ale to pomijamy

$$\Phi'''(\sqrt{R}) = \frac{48R(-9\cdot R^2+18R^2-R^2)}{(3R+R)^4} \neq 0$$

$$\parallel$$

$$\frac{48R^3\cdot(-8)}{(4R)^4} = -\frac{6}{4R}$$

Zatem o ile $R>1$ mamy

$$0 < |\Phi'''(\sqrt{R})| < 1.$$