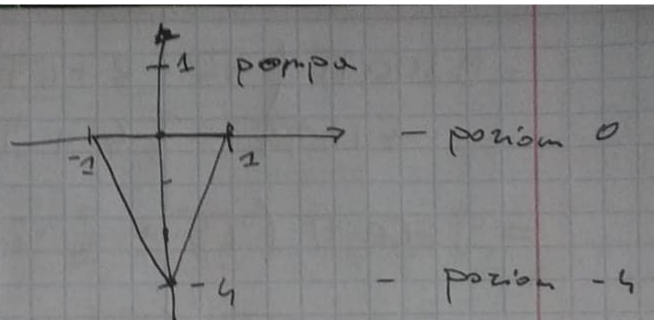
LISTA 7

Zad. 4 4 PKT



$\cancel{A(x) = (\tfrac{1}{4}x + 1)}$

$\cancel{x \in [-4, 0]}$

$R(x) = (\frac{1}{4}x + 1)$

$A(x) = \pi R^2(x)$ $\quad / \; x \in [-4, 0]$

$$W = 9800 \cdot \int_{-4}^{0} (1-x) \cdot \pi (\tfrac{1}{4}x + 1)^2 dx =$$

$$= 9800 \pi \int_{-4}^{0} (\tfrac{1}{16}x^2 + \tfrac{1}{2}x + 1)(1 - x) dx =$$

$$= 9800 \cdot \pi \cdot \int_{-4}^{0} (-\tfrac{1}{16}x^3 - \tfrac{7}{16}x^2 - \tfrac{1}{2}x + 1) dx =$$

$$= 9800 \cdot \pi \cdot \left((-\tfrac{1}{64}x^4 - \tfrac{7}{48}x^3 - \tfrac{1}{4}x^2 + x) \Big|_{-4}^{0} \right) =$$

$$= 9800 \pi \cdot (-(-4 + \tfrac{28}{3} - 4 - 4)) \approx 9800 \cdot 3{,}14 \cdot \tfrac{8}{3} \approx$$

$$\approx 82058 \text{ J}$$

Żeby wypompować połowę wody trzeba wypompować wodę między poziomami ~~0, 2∛4~~ $0, \; 2\sqrt[3]{4} - 4$.

$$W_{\frac{1}{2}} = 9800 \cdot \int_{2\sqrt[3]{4} - 4}^{0} (1-x) \pi (\tfrac{1}{4}x + 1)^2 dx =$$

$$= 9800 \cdot \pi \left((-\tfrac{1}{64}x^4 - \tfrac{7}{48}x^3 - \tfrac{1}{4}x^2 + x) \Big|_{2\sqrt[3]{4} - 4}^{0} \right) =$$

$$= 9800 \cdot \pi \Big(\tfrac{1}{64} \cdot (16 \cdot 4\sqrt[3]{4} - 4 \cdot 8 \cdot 4 \cdot 4 + 6 \cdot 4 \cdot \sqrt[3]{16} \cdot 16 - 4 \cdot 2\sqrt[3]{4} \cdot 64 +$$

$$+ 256) + \tfrac{7}{48}(8 \cdot 4 - 3 \cdot 4\sqrt[3]{16} \cdot 4 + 3 \cdot 2\sqrt[3]{4} \cdot 16 - 64)$$

$$+ \tfrac{1}{4}(4\sqrt[3]{16} - 2 \cdot 2\sqrt[3]{4} \cdot 4 + 16) - (2\sqrt[3]{4} - 4) \Big) =$$

$$= 9800\pi \left(\sqrt[3]{4} - 8 + 6\sqrt[3]{16} - 8\sqrt[3]{4} + 4 + \frac{7}{3}\cdot 2 - \cancel{\sqrt[3]{16}} \right.$$

$$\left. + 14\sqrt[3]{4} - \frac{7}{3}\cdot 4 + \sqrt[3]{16} - 4\sqrt[3]{4} + 4 - 2\sqrt[3]{4} + 4 \right) =$$

$$= 9800\pi \left(\sqrt[3]{4} + \frac{7}{3}\cdot 2 - \frac{7}{3}\cdot 4 + 4 \right) =$$

$$= 9800\pi \cdot \left(\sqrt[3]{4} + 4 - \frac{14}{3} \right) \approx$$

$$9800 \cdot 3.14 \cdot 0.92 \approx 28310 \quad [J]$$

zad. 5 4 PKT

Woda będzie wyciekać przez 20s, więc podniesiemy je na 10m. Niech $M(t) = \frac{1}{2} + (10 - \frac{1}{2}t)$ to masa wiadra w czasie t.

Siła potrzebna do podniesienia wiadra o masie $M(t)$ to oczywiście $M(t)\cdot g$. Praca potrzebna do podniesienia go o s metrów to $M(t)\cdot g \cdot s$.

Podzielmy czas na n części. Niech m_i to „średnia" masa wiadra z wodą w i-tym momencie. Wtedy

$$W \approx \sum_{i=1}^{n} m_i \cdot \underbrace{\underbrace{\frac{1}{n}\cdot 20}_{\text{czas}} \cdot \underbrace{\frac{1}{2}}_{\text{prędkość}}}_{\text{droga}} \cdot m_i \cdot g$$

$$\cancel{W = g\cdot 10 \cdot \int_0^{20} M^2(t)\,dt = 9{,}8\cdot 10 \cdot \int_0^{20} \left(10\tfrac{1}{2} - \tfrac{1}{2}t\right) dt}$$

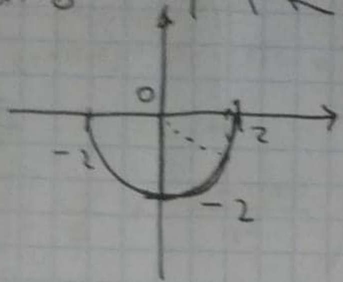
$$W = g\cdot 20 \cdot \frac{1}{2} \int_0^{20} M^2(t)\,dt = 98 \cdot \int_0^{20} \left(10\tfrac{1}{2} - \tfrac{1}{2}t\right)^2 dt =$$

$$= 98 \int_0^{20} \left(\frac{441}{4} - \frac{21}{2}t + \frac{1}{4}t^2 \right) dt = 98\left(\frac{441}{4}t - \frac{21}{4}t^2 + \frac{1}{12}t^3 \right)\Bigg|_0^{20} =$$

$$= 98\left(5\cdot 441 - 21\cdot 20\cdot 5 + 20\cdot 20\cdot 5\cdot \frac{1}{3} \right) =$$

$$= 98 \cdot 771\tfrac{2}{3} = 75623\tfrac{1}{3} \quad [J]$$

zad. 6 4 PKT



$$S(t) = 10\cdot\left(\sqrt{4-t^2}\cdot 2\right)$$

$$W = 9800\cdot\int_{-2}^{0}(0-t)\cdot S(t)\,dt =$$

$$= 9800\int_{-2}^{0} -t\cdot 20\sqrt{4-t^2}\,dt =$$

$$= 2\cdot 98000\int_{-2}^{0} t\sqrt{4-t^2}\,dt = \left|\begin{array}{l} 4-t^2 = z \\ -2t\,dt = dz \end{array}\right| =$$

$$= -98000\cdot 2\cdot\int_{0}^{4} -\frac{1}{2}\sqrt{z}\,dz = -98000\left(-\frac{1}{2}\cdot\frac{2}{3}z^{\frac{3}{2}}\Big|_{0}^{4}\right) =$$

$$= 2\cdot 98000\cdot\frac{1}{3}\cdot 8 \approx 522667 \; [J]$$

zad. 10

a) 4 PKT

Ze względu na symetrię półsfery musimy znaleźć jedynie wsp. z środka masy, gdyż y = x = 0.

$$z = \frac{\sum_{i=0}^{n}\left(\frac{i}{n}\cdot a\right)\cdot\overbrace{2\sqrt{a^2-\left(\frac{ia}{n}\right)^2}\cdot\pi}^{\text{masa}}}{\sum_{i=0}^{n} 2\sqrt{a^2-\left(\frac{ia}{n}\right)^2}\cdot\pi} \longrightarrow \frac{\int_0^a t\, 2\pi\sqrt{a^2-t^2}\,dt}{\int_0^a 2\pi\sqrt{a^2-t^2}\,dt}$$

$$= 2\pi\int_0^a t\sqrt{a^2-t^2}\,dt = \left|\begin{array}{l} a^2-t^2 = u \\ -2t\,dt = du \end{array}\right| = 2\pi\int_{a^2}^{0} -\frac{1}{2}\sqrt{u}\,du =$$

$$= \pi\int_0^{a^2}\sqrt{u}\,du = \pi\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_0^{a^2} = \frac{2}{3}\pi\cdot a^3$$

$$2\pi\int_0^a \sqrt{a^2-t^2}\,dt = \left| \begin{array}{l} t = a\sin u \\ dt = a\cos u\,du \end{array}\right| = 2\pi\int_0^{\frac{\pi}{2}}\sqrt{a^2-(a\sin u)^2}\cdot a\cos u\,du =$$

$$= 2\pi\int_0^{\frac{\pi}{2}} a^2\cos^2 u\,du = 2\pi a^2\cdot\left(\frac{\sin 2u}{4} + \frac{u}{2}\right)\Bigg|_0^{\frac{\pi}{2}} =$$

$$= 2\pi a^2\cdot\left(0+\frac{\pi}{4}\right) = \frac{\pi^2 a^2}{2}$$

$$Z = \frac{\frac{2}{3}\pi\cdot a^3}{\frac{\pi^2 a^2}{2}} = \frac{4a}{3\pi}$$

b) 5 PKT

Podobnie $x=y=0$, trzeba znaleźć

$$z. \quad Z = \frac{\int_0^a t\cdot\pi\left(\sqrt{a^2-t^2}\right)^2dt}{\int_0^a \pi\left(\sqrt{a^2-t^2}\right)^2 dt} =$$

$$= \frac{\int_0^a t(a^2-t^2)\,dt}{\int_0^a (a^2-t^2)\,dt} = \frac{\frac12 a^2t^2 - \frac14 t^4 \Big|_0^a}{a^2t-\frac13t^3\Big|_0^a} =$$

$$= \frac{\frac12 a^4 - \frac14 a^4}{a^3-\frac13 a^3} = \frac{3}{8}a$$