1. Introduction

2. Preliminaries

2.1. Descriptive set theory.

Definition 2.1. Suppose X is a topological space and $A \subseteq X$. We say that A is *meagre* in X if... Wea say that A is *comeagre* in X if... .

Definition 2.2. We say that a topological space X is a *Baire space* if every comeagre subset of X is dense in X (equivalently, every meagre set has empty interior).

Definition 2.3. Suppose X is a Baire space. We say that a property P holds generically for a point in $x \in X$ if $\{x \in X \mid P \text{ holds for } x\}$ is comeagre in X.

Example 2.4. content

2.2. Fraïssé classes.

Fact 2.5 (Fraïssé theorem). *Suppose $\mathscr{C}$ is a class of finitely generated L-structures such that...*

Then there exists a unique up to isomorphism counable L-structure M such that...

Definition 2.6. For $\mathscr{C}$, M as in Fact 2.5, we write $\mathrm{FLim}(\mathscr{C}) := M$.

Fact 2.7. *If $\mathscr{C}$ is a uniformly locally finite Fraïssé class, then $\mathrm{FLim}(\mathscr{C})$ is $\aleph_0$-categorical and has quantifier elimination.*

3. Conjugacy classes in automorphism groups

3.1. Prototype: pure set.

In this section, $M = (M, =)$ is an infinite countable set (with no structure beyond equality).

Proposition 3.1. *If $f_1, f_2 \in \mathrm{Aut}(M)$, then f_1 and f_2 are conjugate if and only if for each $n \in \mathbf{N} \cup \{\aleph_0\}$, f_1 and f_2 have the same number of orbits of size n.*

Proposition 3.2. *The conjugacy class of $f \in \mathrm{Aut}(M)$ is dense if and only if...*

Proposition 3.3. *If $f \in \mathrm{Aut}(M)$ has an infinite orbit, then the conjugacy class of f is meagre.*

Proposition 3.4. *An automorphism f of M is generic if and only if...*

Proof. □

3.2. More general structures.

Proposition 3.5. *Suppose M is an arbitrary structure and $f_1, f_2 \in \mathrm{Aut}(M)$. Then f_1 and f_2 are conjugate if and only if $(M, f_1) \cong (M, f_2)$.*

Definition 3.6. We say that a Fraïssé class $\mathscr{C}$ has *weak Hrushovski property* (*WHP*) if for every $A \in \mathscr{C}$ and partial automorphism $p : A \to A$, there is some $B \in \mathscr{C}$ such that p can be extended to an automorphism of B, i.e. there is an embedding $i : A \to B$ and a $\bar{p} \in \mathrm{Aut}(B)$ such that the following diagram commutes:

$$\begin{array}{ccc} B & \xrightarrow{\bar{p}} & B \\ {\scriptstyle i}\uparrow & & {\scriptstyle i}\uparrow \\ A & \xrightarrow{p} & A \end{array}$$

Proposition 3.7. *Suppose $\mathscr{C}$ is a Fraïssé class in a relational language with WHP. Then generically, for an $f \in \mathrm{Aut}(\mathrm{FLim}(\mathscr{C}))$, all orbits of f are finite.*

Proposition 3.8. *Suppose $\mathscr{C}$ is a Fraïssé class in an arbitrary countable language with WHP. Then generically, for an $f \in \mathrm{Aut}(\mathrm{FLim}(\mathscr{C}))$ ...*

3.3. **Random graph.**

Definition 3.9. The *random graph* is...

Fact 3.10. *The*

Proposition 3.11. *Generically, the set of fixed points of $f \in \mathrm{Aut}(M)$ is isomorphic to M (as a graph).*